Lec 5: Adverse Selection

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(⊛) Suggested readings: Mas-Colell et al. (1995), Ch13.A–B.

1 Types of Uncertainty

- Adverse Selection (A.S.): hidden information; uncertain about payoff characteristics
- Moral Hazard (M.H.): hidden action; uncertain about other's actions

Question. Are the following statements true?

① Whenever there exists "gains from trade," the agents prefers to trade.

② (Law of Supply). Lowering prices will increases sales.

③ Reducing the uncertainty about the object value will increase the # of trades.

Answer. ①, ②, ③ are all **NOT** true. (**Why?**)

2 Market for "Lemons" (Akerlof, 1970)

Model 2.1 (Akerlof 1970)**.** Consider a used-car market with heterog. quality of car.

- **Player**: 1 Buyer (**B**), 1 Seller (**S**)
- **Valuation**: $\begin{cases} B : B_i = v_B\theta \\ S : S_i = v_S\theta \end{cases}$, where $\theta \sim U[0, 1]$: quality of a car
- **Payoff**: $\begin{cases} u_B = v_B\theta - p \\ u_S = p - v_S\theta \end{cases}$. Assume $v_B > v_S > 0$.

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WANT. Given Total Surplus: $TS = u_B + u_S = (v_B - v_S)\theta$, we want to know:

- Efficient allocation (will the market delivers efficient allocation?)
- Efficient market p (can we find a price so that we reach efficient outcome?)

$\longrightarrow$ First note that the social optimality is "to trade," regardless of θ and p, since:

$$TS = (v_B - v_S)\theta \geq 0 \tag{2.1}$$

The reason why p does NOT matter is that p serves just as a transfer payment.
$\longrightarrow$ Actually, p depends on **information structure**. Consider the following stages:

Exercise 2.1 (Ex-ante). Neither **B** nor **S** knows the exact quality of car (θ), but knows $\theta \sim U[0,1]$. The participation/**individual rationality (IR)** constraints yield:

$$\text{(IR)}: \begin{cases} u_B \geq 0 \implies \mathbb{E}[v_B\theta - p] = v_B\mathbb{E}[\theta] - p = \frac{1}{2}v_B - p \geq 0 \\ u_S \geq 0 \implies \mathbb{E}[p - v_S\theta] = p - v_S\mathbb{E}[\theta] = p - \frac{1}{2}v_S \geq 0 \end{cases} \tag{2.2}$$

$\implies$ At *ex-ante* stage, $p \in \left[\frac{1}{2}v_S, \frac{1}{2}v_B\right]$ ensures trades happen. □

Exercise 2.2 (Ex-post). Both **B** and **S** knows the exact realization of θ. The **(IR)** constraints yield:

$$\text{(IR)}: \begin{cases} u_B \geq 0 \implies v_B\theta - p \geq 0 \\ u_S \geq 0 \implies p - v_S\theta \geq 0 \end{cases} \tag{2.3}$$

$\implies$ At *ex-post* stage, $p \in [v_S\theta, v_B\theta]$ ensures trades happen. □

Remark. Notice that p at Ex-post depends on θ. This reflects the bilateral trade as both parties know θ and may bargain the price given θ.

Remark. The Ex-ante & Ex-post stages are pretty straightforward. However, things become somewhat complicated at *interim stage* (see next Example 2.3).

Exercise 2.3 (Interim). Only **S** knows the quality θ. **B** does not know the exact θ but knows $\theta \sim U[0,1]$. We show it by **backward induction**.
① First, the **(IR)** constraint of **S** yields:

$$(\text{IR}; \mathbf{S}) : u_S \geq 0 \implies p - v_S\theta \geq 0 \implies \theta \leq \frac{p}{v_S} \tag{2.4}$$

② Expecting this "signal," the **(IR)** constraint of **B** has expected payoff:

$$(\text{IR}; \mathbf{B}) : \mathbb{E}\left[u_B \mid \theta \leq \frac{p}{v_S}\right] \geq 0 \implies v_B\mathbb{E}\left[\theta \mid \theta \leq \frac{p}{v_S}\right] - p \geq 0 \tag{2.5}$$

$$\iff v_B\left(\frac{1}{\frac{p}{v_S}}\int_0^{\frac{p}{v_S}} \theta d\theta\right) - p \geq 0 \tag{2.6}$$

$$\iff v_B\left(\frac{p}{2v_S}\right) - p \geq 0 \tag{2.7}$$

$$\iff v_B \geq 2v_S \tag{2.8}$$

$\implies$ At *interim* stage, trades happen only when $v_B \geq 2v_S$. (★)

Remark. At interim stage, p signals some information about car quality (see Eq (2.4)).

Summary (Akerlof's)**.** Now let's revisit our **Questions**:

① Whenever there exists "gains from trade," the agents prefers to trade.
$\implies$ **No.** Even if "to trade" is socially optimal, trades occur only when $v_B \geq 2v_S$!

② (Law of Supply). Lowering prices will increases sales.
$\implies$ **No.** Lowering p only signals the "worse" quality of a car by $\mathbb{E}[\theta|\theta \leq \frac{p}{v_S}] = \frac{p}{2v_S}$!

③ Reducing the uncertainty about the object value will increase the # of trades.
$\implies$ **No.** At Ex-ante, the best is to always trade, but at Interim trades happen only if $v_B \geq 2v_S$. We reveal more information from Ex-ante to Interim, but the trade occurrence shrinks!

$\longrightarrow$ **Why?** There is **asymmetric information** in valuation.

3 Prices as Signals of Product Quality (Wolinsky, 1983)

Question. Consider the following Motivating Questions:

- How does asymmetric info about quality affect efficiency?
- What is the role of the "informed" agent in guaranteeing the trade?

Model 3.1 (Wolensky 1983)**.** Consider a monopoly market by:

- **Player**: 1 Seller (**S**), a unit mass of Buyers (**B**).
- **Quality**: **S** can produce type $\{H, L\}$ quality of cars, with:

$$v_H - c_H > v_L - c_L \quad (3.1)$$

$$c_H - c_L > 0 \quad (3.2)$$

  where producing H is more efficient.

- **S** chooses price p & quantity q simultaneously. **B** knows quantity.
- **Informed buyers**: a portion of α buyers are informed of p, $(1-\alpha)$ buyers uninformed.
- **WANT.** Incentivise **S** to produce cars of H quality.

Let's analyze the case buyer buys H and seller *only* produces H:

① **(IR)** constraints of **B**:

$$(\text{IR; } \mathbf{B}) : \begin{cases} \text{Informed } \mathbf{B} \text{ buys a car with quality H if } v_H - p \geq 0 \\ \text{Uninformed } \mathbf{B} \text{ buys a car if } \mathbb{E}[v|p] - p \geq 0 \end{cases} \quad (3.3)$$

② **S** produces H when her **(IR)** & **(IC)** (Incentive Compatibility) constraints are satisfied:

$$\mathbf{S} : \begin{cases} \textbf{(IC; S)} : 1 \cdot (p - c_H) \geq (1-\alpha)(p - c_L) \\ \textbf{(IR; S)} : p - c_H \geq 0 \end{cases} \quad (3.4)$$

⊛ *I think of **(IC; S)** as "all **B** buys H" when only producing H (hence times* 1*) versus the original scenario where at least uninformed **B** buys L and informed **B** buys nothing.*

- Case: $\alpha = 0$ (All **B** uninformed) Then we know **S** will only produce L (not H) since **(IC; S)** $\implies p - c_H \geq p - c_L \implies c_H \leq c_L$, a contradiction to Eq (3.2)!
- Case: $\alpha > 0$ (Some **B** informed) Then, by **(IC; S)**:

$$p - c_H \geq p - \alpha p - (1-\alpha)c_L \implies p \geq \frac{c_H - (1-\alpha)c_L}{\alpha} \; (\bigstar) \quad (3.5)$$

  ⊛ If $\alpha \to 0$: then $p \to \infty \implies$ **(IR; B)** never holds $\implies$ No trade!

  ⊛ If $\alpha \to 1$: then $p \geq c_H \implies$ **(IR; S)** & **(IC; S)** both (✓) $\implies$ Need: **(IR; B)**

  ⊛ If α suff. large s.t. ($\bigstar$) holds $\implies$ the allocation is efficient! (✓)

Summary (Wolensky's)**.** What have we learned from this Adverse Selection model?

- The production of H is induced by suff. large p which satisfy ($\bigstar$).
- p indeed signals quality (i.e., tell some information to uninformed agent, which is buyer)
- Role of informed agent (α): A higher α induces more incentive for seller to produce goods of H quality.

4 Exercise from DIS SEC

Exercise 4.1 (Market for Lemons)**.** Consider a unitary mass of **S** of used cars. Each **S** has exactly one used car to sell and is characterized by their quality. Let $\theta \sim U[0,1]$ index the quality of a used car. The utility of a **S** of a car quality θ at price p is $u_S(p,\theta)$. Meanwhile, a unitary mass of **B** have linear utility $u_B(p,\theta) = \theta - p$ if the buy the car and zero otherwise.

(a) Argue that in a COMP EQM under asymmetric information, we have $\mathbb{E}[\theta|p] = p$.

(b) Show that if $u_S(p,\theta) = p - \frac{\theta}{2}$, then every $p \in (0, \frac{1}{2}]$ is an EQM price.

(c) Describe the equilibrium for $u_S(p,\theta) = p - \sqrt{\theta}$.

(d) Describe the equilibrium for $u_S(p,\theta) = p - \theta^3$.

Solution (a)**.** We must have $\mathbb{E}[\theta|p] = p$ in CE since:

- If $\mathbb{E}[\theta|p] < p \implies \mathbb{E}[u_B(p,\theta)] = \mathbb{E}[\theta|p] - p < 0$: **no trade!**
- If $\mathbb{E}[\theta|p] > p \implies \mathbb{E}[u_B(p,\theta)] = \mathbb{E}[\theta|p] - p > 0$: p can $\uparrow$ without $\downarrow$ demand, **not CE!**

Solution (b)**.** Suppose now $u_S(p,\theta) = p - \frac{\theta}{2}$, then the marginal **S** has:

$$0 \le u_S(p,\theta) = p - \frac{\theta}{2} \implies \theta \le 2p \tag{4.1}$$

So, **B** expects this and have:

$$\begin{aligned}
\mathbb{E}[u_B(p,\theta)|p] &= \mathbb{E}[\theta - p|p] && (4.2)\\
&= \mathbb{E}[\theta|\theta \le 2p] - p && (4.3)\\
&= \left[\frac{1}{2p}\int_0^{2p} \theta d\theta\right] - p && (4.4)\\
&= \frac{1}{2}(2p) - p = 0 \longleftarrow \text{all } \mathbf{B} \text{ buys} && (4.5)
\end{aligned}$$

Since $\theta \in [0,1]$, we have $p \in [0, \frac{1}{2}]$. But we requires $p \neq 0$ at Eq (4.4) $\implies p \in (0, \frac{1}{2}]$. We conclude that a portion of $2p$ **S** will sell s.t. $2p$ of **B** get to buy and the rest $(1-2p)$ don't.

Solution (c). Suppose now $u_S(p,\theta) = p - \sqrt{\theta}$, then similarly we consider the marginal **S**:

$$0 \le u_S(p,\theta) = p - \sqrt{\theta} \implies \theta \le p^2 \tag{4.6}$$

So, **B** expects this and have:

$$\begin{aligned}
\mathbb{E}[u_B(p,\theta)|p] &= \mathbb{E}[\theta - p|p] = \mathbb{E}[\theta|\theta \le p^2] - p && (4.7)\\
&= \left[\frac{1}{p^2}\int_0^{p^2} \theta d\theta\right] - p = \frac{1}{2}(p^2) - p && (4.8)\\
&= \frac{1}{2}p\underbrace{(p-1)}_{<0} < 0 \longleftarrow \text{NO } \mathbf{B} \text{ buys} && (4.9)
\end{aligned}$$

We conclude that there is no demand at all $\implies$ **NO trade!**

Solution (d). Suppose now $u_S(p,\theta) = p - \theta^3$, then similarly we consider the marginal **S**:

$$0 \le u_S(p,\theta) = p - \theta^3 \implies \theta \le p^{\frac{1}{3}} \tag{4.10}$$

So, **B** expects this and have:

$$\begin{aligned}
\mathbb{E}[u_B(p,\theta)|p] &= \mathbb{E}[\theta - p|p] = \mathbb{E}[\theta|\theta \le p^{\frac{1}{3}}] - p && (4.11)\\
&= \left[\frac{1}{p^{\frac{1}{3}}}\int_0^{p^{\frac{1}{3}}} \theta d\theta\right] - p = \frac{1}{2}(p^{\frac{1}{3}}) - p \ (\spadesuit) && (4.12)
\end{aligned}$$

Let's solve $(\spadesuit) = 0$ for the EQM # of trades:

$$\frac{1}{2}(p^{\frac{1}{3}}) - p = 0 \implies \frac{1}{8}p = p^3 \implies p = \frac{1}{2\sqrt{2}} \tag{4.13}$$

We conclude that a $\frac{1}{2\sqrt{2}}$ portion of cars are traded in the market.

References

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